Continuous Spectrum of a Flange-Backed Slotted Waveguide with Application

Tullio Rozzi, *Fellow, IEEE*, and Mauro Mongiardo

***Abstract*—The continuum of an open waveguide describes radiation as simply, in principle, as the discrete spectrum of a classical guide describes any physical field in it. This part of the spectrum, however, has received little attention for guides of nonseparable two-dimensional cross-sections. To illustrate its derivation, in this contribution we establish the continuum of a flange-backed rectangular waveguide slotted on its narrow wall. As a demonstration for its use, we determine the transmission and radiation properties of the junction between an ordinary and slotted guide.**

Introduction

THE CLASSICAL method for determining the e.m. field excited by a source in any closed waveguide is to evaluate the modal amplitudes excited by the source by means of Lorentz theorem or its equivalents [1, pp. 358–362].

The same process can be followed in any open guide of one-dimensional cross-section, say, a dielectric slab [2, pp. 303–306, 1, pp. 485–495, pp. 538–546, 3], or a two-dimensional separable one, say, an optical fiber. The power distributes itself among bound modes, if any, and a continuous orthonormal spectrum of real waves, bound at infinity. The proper spectrum of the guide is not to be confused with the "leaky modes" that are nonmodal, nonorthogonal, complex solutions of the wave equation (of the transverse resonance condition), growing at infinity and, as such, unsuitable for representing the field except in the immediate vicinity of the source.

For most guides of two-dimensional and nonseparable cross-section, however, including, the classical slotted waveguide, the continuous spectrum is not known. Earlier contributions on the pure LSE/LSM continua of the inset [4] and image [5] guide utilized a partial wave (spectral) decomposition of the field in a transverse direction of the guide cross-section. Each partial wave underwent a different phase-shift according to its transverse wavenumber due to transverse diffraction and the total field was then recomposed by superposition.

In this scheme, each partial wave did not individually satisfy boundary and edge conditions in the transverse cross-section, resulting in various drawbacks. In [6] the principle of a new approach was developed; this deal with the fully hybrid case. It introduced the concept of a continuum of mutually orthonormal "packets" of waves, each packet traveling with the same propagation constant along the guide, each individually satisfying boundary and edge conditions in the cross-section. The latter formulation, being essentially self-consistent with transverse diffraction, is proving superior to the earlier one.

In the present contribution, we will reconsider the classical problem of rectangular waveguide slotted in its narrow wall, that was studied many years ago by various authors as a "leaky" guide antenna and is described, for instance, in [7], [8]. The simplicity of the geometry allows insight in the operation of the new method with a minimum of analytical detail.

In this case, just a continuous spectrum exists; once this is derived, relying on [6] for general proofs, we demonstrate its application to a practical problem by applying it to the problem of determining transmission and radiation properties of the junction between a flange-backed ordinary guide and one slotted on its narrow wall.

Analysis

The geometry under study is shown in Fig. 1. It consists of a classical rectangular waveguide with a slot symmetrically placed on its narrow-wall (of zero thickness) backed by a perfectly conducting flange.

If the excitation is a pure TE, i.e., by the fundamental waveguide mode, this problem is describable in terms of a single TE potential $\underline{\Pi}_h = z_o\Psi_h$. Clearly, no bound solutions are possible for this problem due to the presence of the slot. We are looking for real solutions of the scalar wave equation for E_y, say,

$$\nabla_t^2 e_\nu + k_t^2 e_\nu = 0 \tag{1}$$

where $k_t^2 = k_x^2 + k_y^2 = k_o^2 - \beta^2$ $(0 \le k_t \le \infty)$ is the transverse wavenumber and ν labels different modes corresponding to the same k_t; e_ν satisfies boundary and edge condition pertaining to E_y on the cross-section S, which comprises that of the guide and the half-space $x \geq 0$ and is finite at infinity. Moreover, the following orthonormalization is imposed:

$$\int_S e_\nu(\underline{r};\, k_t)\, e_\mu(\underline{r};\, k_t')\, dS = \delta_{\nu\mu}\,\delta(k_t - k_t') \tag{2}$$

with $\underline{r} = (x, y)$.

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T. Rozzi is with the Dipartimento di Elettronica, ed Automatica, Università degli Studi di Ancona, Via Brecce Bianche, 60131 Ancona, Italy.
M. Mogiardo is with the Dipartimento di Ingegneria Elettronica, Università di Roma "Tor Vergata," Rome, Italy.
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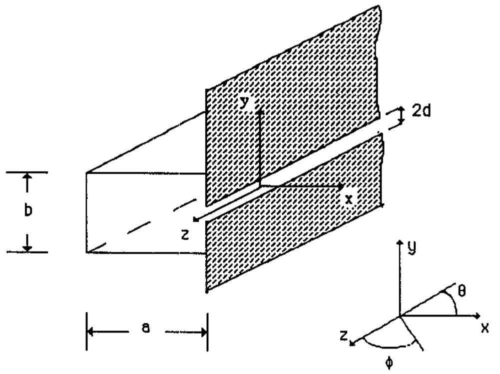

Fig. 1. Geometry of a flange-backed rectangular waveguide with a symmetrical slot on its narrow wall.

The field we seek, e_ν, has different forms in the waveguide and in the air region and each is made up of a superposition of plane waves such that $k_x^2 + k_y^2 = k_t^2$. The form that holds in the waveguide is of the type

$$e_\nu(x, y; k_t) = N_\nu(k_t) \sum_{n=0,2}^{\infty} \tilde{E}_\nu n(k) c_n(y) \frac{\sin q_n(x+a)}{\sin q_n a}: \quad x < 0. \tag{3a}$$

$N_\nu(k_t)$ is a normalization constant, $\tilde{E}_{\nu n}$ is the discrete Fourier transform of the field in the slot pertaining to the mode ν for that value of k_t; of course this field is identically zero on the flange plane. Moreover

$$c_n(y) = \begin{Bmatrix} \sqrt{\frac{2}{b}} \cos \frac{n\pi y}{b} & n = 2, 4, \cdots \\ \sqrt{\frac{1}{b}} & n = 0 \end{Bmatrix}$$

in view of the symmetrical location of the iris on the sidewall and

$$q_n^2 = k_t^2 - \left(\frac{n\pi}{b}\right)^2.$$

Form (3a) is the field excited in the guide by a given aperture distribution $E_\nu(y; k_t)$. We will now write the corresponding form of the field excited in the air half-space as

$$e_\nu(x, y; k_t) = N_\nu(k_t) \left[\int_0^{k_t} \tilde{E}_\nu(k_y; k_t) c(y; k_y) \cdot \frac{\sin(k_x x + \alpha_\nu(k_t))}{\sin \alpha_\nu(k_t)} dk_y + \int_{k_t}^{\infty} \tilde{E}_\nu(k_y; k_t) c(y; k_y) e^{-|k_x| x} dk_y \right] \tag{3b}$$

where

$$c(y; k_y) = \sqrt{\frac{2}{\pi}} \cos k_y y$$

is the continuous analog of $c_n(y)$; $k_x^2 + k_y^2 = k_t^2$ and

$$\tilde{E}(k_y; k_t) = \int_{-d}^{d} E_\nu(y; k_t) c(y; k_y)\, dy = \langle c, E_\nu \rangle. \tag{4}$$

The above Fourier transform is taken just on the slot as the field vanishes outside the slot on the flange plane.

In (3b) we distinguish two types of behavior in x of the plane waves components:

i) $k_y < k_t$: standing waves in x in order to describe the radiation incident from infinity on the slotted flange and reflected back from it. In this context, the real quantity $\alpha_\nu(k_t)$ assumes the physical meaning of the phase shift imposed on the incident $e^{jk_x x}$ component by the presence of the slot and of the waveguide behind it. $\sin(\alpha_\nu) = 0$ corresponds to the flange being completely closed. Note that the process of placing the generator at infinity and considering standing waves is identical to that followed in deriving the continuum of a slab waveguide [2].

ii) $k_y > k_t$: in this case only attenuated waves away from the slot can exist in the air region.

Upon inspection of (3) we note that $e_\nu(k_t)$ is completely defined once we have determined the field on the aperture $E_\nu(y; k_t)$, the phase shift $\alpha_\nu(k_t)$ and the normalization constant $N_\nu(k_t)$.

We shall now proceed to establish an eigenvalue equation that has $E_\nu(y; k_t)$ as eigenfunction and α_ν as eigenvalue. This is obtained by requiring the continuity of the H_z component, $h_\nu(x, y; k_t)$ corresponding to $e_\nu(x, y; k_t)$.

Modal TE fields can be derived from TE potentials

$$\begin{aligned} \underline{\Pi}_\nu &= \underline{z}_o \Psi_\nu e^{-j\beta z} \\ \underline{h}_{t\nu} &= -j\beta \nabla_t \Psi_\nu \\ \underline{e}_{t\nu} &= -\frac{\omega\mu_0}{\beta} \underline{z}_0 \times \underline{h}_{t\nu} \\ e_\nu &= y_0 \cdot \underline{e}_{t\nu} \\ h_\nu &= k_t^2 \Psi_\nu. \end{aligned} \tag{5}$$

Hence the relationship valid everywhere in the cross-section is

$$h_\nu = \frac{k_t^2}{j\omega\mu_0} \int e_\nu\, dx. \tag{6}$$

It is noted that a vanishing integration constant is required in order to fulfill the boundary condition on the slot edge and at infinity.

By substituting (3) into (6) and setting $x = 0$ we obtain the following eigenvalue equation for $\alpha_\nu(k_t)$

$$\cot \alpha_\nu(k_t) \int_0^{k_t} \tilde{E}_\nu(k_y, k_t) \frac{c(y; k_y)}{k_x} dk_y = \sum_{n=0}^{\infty} \tilde{E}_{\nu n}(k_t) c_n(y) \frac{\cot q_n a}{q_n} - \int_{k_t}^{\infty} \tilde{E}_\nu(k_y, k_t) \frac{c(y; k_y)}{|k_x|} dk_y. \tag{7}$$

Its eigenfunction is the aperture distribution $E_\nu(y;\, k_t)$ whose Fourier transform appears in (7).

If we introduce the total transverse admittance operator for this cross-section, comprising the waveguide and the air region,

$$\hat{Y}(k_t) = \hat{Y}_{\text{air}}(k_t) + \hat{Y}_{wg}(k_t) \tag{8}$$

with

$$\hat{Y}_{wg}(k_t)E_\nu \equiv \sum_{n=0}^{\infty} -j\,\langle E_\nu,\, c_n\rangle\, c_n(y)\, \frac{\cot q_n a}{q_n}$$

$$\hat{Y}_{\text{air}}(k_t)E_\nu \equiv \int_0^{k_t} \langle E_\nu,\, c\rangle\, \frac{c(y;\, k_y)}{k_x}\, dk_y + j\int_{k_t}^{\infty} \langle E_\nu,\, c\rangle\, \frac{c(y;\, k_y)}{|k_x|}\, dk_y \tag{9}$$

the eigenvalue equation (7) can be rewritten as

$$\cot \alpha_\nu(k_t)\, \text{Re}\, \hat{Y}(k_t)E_\nu = \text{Im}\, \hat{Y}(k_t)E_\nu \tag{10}$$

with

$$\hat{Y} = \text{Re}\, \hat{Y} - j\,\text{Im}\, \hat{Y}$$

or, equivalently,

$$\hat{Y}(k_t)E_\nu = \lambda_\nu\, \text{Re}\, \hat{Y}(k_t)E_\nu \tag{11}$$

with

$$\lambda_\nu = 1 - j\cot \alpha_\nu.$$

If Re $\hat{Y}(k_t)$ vanishes, say, i.e., in absence of radiation, then the standard transverse resonance condition is recovered, yielding the bound modes of the system. No such solution is possible in this context, only solutions of the type (3) with normalization (2) are proper, modal, solutions.

From (10) or (11) it is easy to see that two solutions E_ν, E_μ corresponding to the same value of k_t are orthogonal over the aperture as

$$\langle E_\mu,\, \text{Re}\, \hat{Y}E_\nu\rangle = \delta_{\mu\nu} \tag{12}$$

implying these two solutions do not exchange power due to the aperture.

By multiplying $e_\nu(k_t)$, $e_\mu(k_t')$ by and integrating over the cross section, under the condition that (10) holds we recover the normalization condition (2) with

$$N_\nu(k_t) = \sqrt{\frac{2k_t}{\pi}}\, \sin \alpha_\nu(k_t). \tag{13}$$

Discretization of the Eigenvalue Equation

An effective discretization of (10) into an ordinary matrix eigenvalue problem is a key step in the process.

We build in the edge condition by setting, for a symmetrically placed aperture,

$$E_\nu(y;\, k_t) = w(y) \sum_{m=0,2}^{N} \sqrt{\frac{\epsilon_m}{\pi d}}\, \chi_m T_m\left(\frac{y}{d}\right)$$
$$\epsilon_0 = 1;\ \epsilon_m = 2:\ m > 0 \tag{14}$$

where χ_m are the expansion coefficients and

$$w(y) = \frac{1}{\sqrt{1 - \left(\frac{y}{d}\right)^2}}$$

and the T_m's are the Chebyshev polynomials normalized so that [9, p. 833]

$$\frac{\sqrt{\epsilon_m\epsilon_n}}{\pi d}\langle T_m,\, wT_n\rangle = \frac{\sqrt{\epsilon_m\epsilon_n}}{\pi d}\int_{-d}^{d} T_m w T_n\, dy = \delta_{mn}. \tag{15}$$

With this position, one substitutes to $\hat{Y}$ the discrete matrices

$$\text{Im}\, Y_{mk} = \frac{k_t^2}{\omega\mu_0}\left[\sum_{n=0,2}^{\infty} \frac{\cot q_n a}{\sqrt{k_t^2 - \left(\frac{n\pi}{b}\right)^2}} P_{mn}P_{kn} - \int_{k_t}^{\infty} \frac{P_m(k_y)P_k(k_y)}{\sqrt{k_y^2 - k_t^2}}\, dk_y\right] \tag{16}$$

$$\text{Re}\, Y_{mk} = \frac{k_t^2}{\omega\mu_0}\int_0^{k_t} \frac{P_m(k_y)P_k(k_y)}{\sqrt{k_t^2 - k_y^2}}\, dk_y \tag{17}$$

where [9, p. 836]

$$P_m(k_y) = \left\langle \sqrt{\frac{\epsilon_m}{\pi d}}\, T_m w,\, \sqrt{\frac{2}{\pi}}\cos k_y y \right\rangle = \sqrt{2\epsilon_m d}\,(-1)^{(m/2)} J_m(k_y d)$$

$$P_{mn} = \left\langle \sqrt{\frac{\epsilon_m}{\pi d}}\, T_m w,\, c_n \right\rangle = \sqrt{\frac{\epsilon_m\epsilon_n \pi d}{b}}\,(-1)^{(m/2)} J_m\left(\frac{n\pi d}{b}\right)$$

and (10) becomes a matrix eigenvalue equation of order $(N + 1)/2$, capable of producing up to $(N + 1)/2$ eigenvectors.

As a particular case, consider the "small aperture" approximation, where

$$E_\nu(y;\, k_t) \approx w(y)\sqrt{\frac{1}{\pi d}} \qquad (T_0 = 1) \tag{18}$$

independently of k_t and ν (10) becomes a scalar equation and we have

$$\cot \alpha(k_t) = \frac{\operatorname{Im} Y_{00}}{\operatorname{Re} Y_{00}} = \frac{\sum_{n=0,2}^{\infty} \frac{\cot q_n a}{\sqrt{k_t^2 - \left(\frac{n\pi}{b}\right)^2}} P_{0n}^2 - \int_{k_t}^{\infty} \frac{P_0^2(k_y)}{\sqrt{k_y^2 - k_t^2}} dk_y}{\int_0^{k_t} \frac{P_0^2(k_y)}{\sqrt{k_t^2 - k_y^2}} dk_y} \qquad k_t \neq \frac{n\pi}{b} \tag{19}$$

the corresponding field outside and inside the guide is given by

$$e(\underline{r}; k_t) = \sqrt{\frac{2k_t d}{\pi}} \left[\int_0^{k_t} dk_y \sqrt{\frac{2}{\pi}} \cos k_y y J_0(k_y d) \sin(k_x x + \alpha) \right] + \sqrt{\frac{2k_t d}{\pi}} \left[\sin \alpha \int_{k_t}^{\infty} dk_y \sqrt{\frac{2}{\pi}} \cos k_y y J_0(k_y d) \exp(-|k_x| x) \right]: \quad x \geq 0$$

$$e(\underline{r}; k_t) = \sqrt{\frac{2k_t d}{b}} \sin \alpha \sum_{n=0,2}^{\infty} J_0\left(\frac{n\pi d}{b}\right) \frac{\sin q_n(x+a)}{\sin q_n a} c_n(y): \quad x \leq 0 \tag{20}$$

It is noted that the modes given by (20) are independent of frequency and only dependent on the parameter k_t.

Application to a Slotted Guide Excited by Waveguide

With reference to Fig. 2, we consider the problem of the junction between a flange-backed ordinary waveguide and one slotted as shown in Fig. 1; whose spectrum has been derived in the previous section.

We neglect higher order mode excitation in the closed guide as well as excitation of the open region for $z < 0$.

At $z = 0$, the E_y component of the incident TE_{10} waveguide mode can be decomposed in terms of the continuous spectrum of the slotted guide

$$e_{y10} = \sqrt{\frac{2}{ab}} \sin \frac{\pi x}{a} = \int_0^{\infty} A(k_t) e(x, y, k_t)\, dk_t \tag{21}$$

with

$$A(k_t) = \int_{-a}^{0} dx \int_{-(b/2)}^{b/2} dy \sqrt{\frac{2}{ab}} \sin \frac{\pi x}{a} e(x, y; k_t)$$

$$= \sqrt{\frac{2k_t}{\pi}} \sin \alpha_\nu \sum_{n=0,2,\cdots}^{\infty} \tilde{E}_n(k_t) \int_{-a}^{0} \sqrt{\frac{2}{a}} \sin \frac{\pi x}{a} \frac{\sin q_n(x+a)}{\sin q_n a} dx \int_{-(b/2)}^{b/2} \sqrt{\frac{1}{b}} c_n(y)\, dy$$

and, by using in the above the appropriate expression for the transform of the field (18) over the aperture

$$\tilde{E}_n = \sqrt{\frac{2}{\pi d}} \int_{-d}^{d} \sqrt{\frac{2\epsilon_n}{b}} \frac{\cos \frac{n\pi y}{b}}{\sqrt{1 - \left(\frac{y}{d}\right)^2}} dy = \sqrt{\frac{\epsilon_n \pi d}{b}} J_0\left(\frac{n\pi d}{b}\right)$$

we get

$$A(k_t) = 2\pi \sqrt{k_t a \frac{d}{b}} \frac{\sin \alpha_v}{(k_t a)^2 - \pi^2}. \tag{22}$$

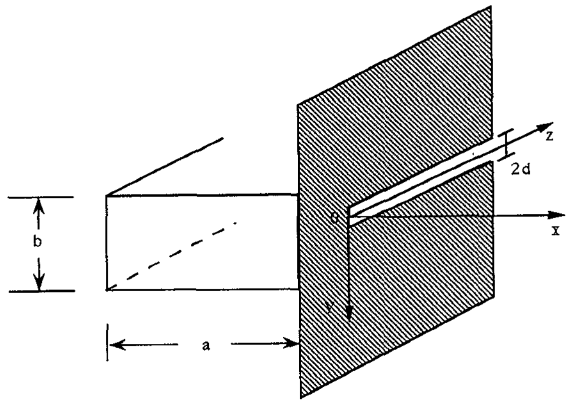

Fig. 2. Geometry of the transition between a closed and a slotted waveguide.

The E_y field at a distance z down the guide is then given by

$$E_y(\underline{r}, k_0) = \int_0^{k_0} dk_t A(k_t) e(\underline{r}, k_t) \exp -jz\sqrt{k_0^2 - k_t^2} + \int_{k_0}^{\infty} dk_t A(k_t) e(\underline{r}, k_t) \exp -z\sqrt{k_t^2 - k_0^2} \quad (23)$$

Numerical Results

The behavior of sin α as obtained from (19) is reported in Fig. 3. It should be noted that α depends only on the guide aspect ratio a/b, relative slot aperture d/a, and on $k_t a$. In this case $d/a = 1/22.86$ As previously stated, the real quantity α represents the phase shift imposed on the incident $e^{jk_x x}$ component by the presence of the slot and of the waveguide behind it. When k_t is zero, that is the wave is directed along z, also sin α is zero. Moreover, when waveguide resonances occur, that is when k_t is equal to the cutoff frequencies of the unperturbed guide, sin α is again zero. These locations are given in Table I.

Once α has been determined it is possible to calculate the modal fields inside and outside the guide according to (20). The behavior of the electric field E_y inside the waveguide is reported in Fig. 4 for different values of k_t and for an aperture $d/a = 1/22.86$. Due to the symmetry of the structure only the upper half-plane ($y > 0$) has been considered. It is possible to observe how the aperture, and the metallic edge, affect the field distribution inside the guide.

In Fig. 5 the electric field E_y in the air region is plotted for the same values of $k_t a$ in the proximity of the guide. The oscillatory behavior of the field in the half space allows (2) to be satisfied even for small differences between k_t and k_t'. Also evident is the creation of a spherical wave at a certain distance from the aperture as illustrated by level curves in Fig. 6.

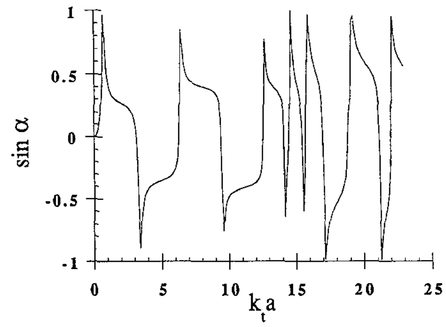

Fig. 3. Behavior of the phase shift α as a function of $k_t a$ for a slotted waveguide. The zeros of sin α correspond to the resonances of the guide reported in Table I.

TABLE I
WAVEGUIDE RESONANCES OBTAINED BY USING

$$k_t a = \pi\sqrt{m^2 + \left(\frac{na}{b}\right)^2}$$

(THESE VALUES CORRESPONDS TO THE ZEROS OF FIG. 3 WHERE $a/b = 2.25$)

m	$n = 0$	$n = 2$
1	3.14	14.48
2	6.28	15.47
3	9.42	16.99
4	12.57	18.91
5	15.71	21.13
6	18.85	23.56
7	21.99	26.14

When the slotted waveguide is excited from a closed one, the incident field is decomposed in terms of the continuous spectrum of the slotted waveguide. Each component of the continuous spectrum is excited with different amplitude, as given from (22). Fig. 7 shows the amplitude of these components, $A(k_t)$, when the fundamental mode is incident on a slotted guide with $d/a = 1/22.86$.

At any given frequency each mode, characterized by a k_t value, has a propagation constant β either real (propagating mode) or imaginary (evanescent mode). For a fixed k_t the angle with the z-axis formed by the direction of propagation of the mode in the air region is given by $\phi = \text{arctg}\,(k_t/\beta)$, while $A(k_t)$ determines its amplitude. A plot relating the latter quantity to the angle ϕ is shown in Fig. 8 for different frequencies.

As expected, as the frequency increases the maximum modal amplitude moves toward endfire. This can be explained by recalling the interpretation of the field inside the waveguide as a superposition of two plane waves. Each plane wave impinges on the narrow wall at an angle with respect to the z direction that increases as the frequency decreases.

It is obvious from the preceding figures the appearance of a "leaky wave" corresponding to a peak of $A(k_t)$ in Fig. 7 and to a preferred angle of radiation as shown in Fig. 8.

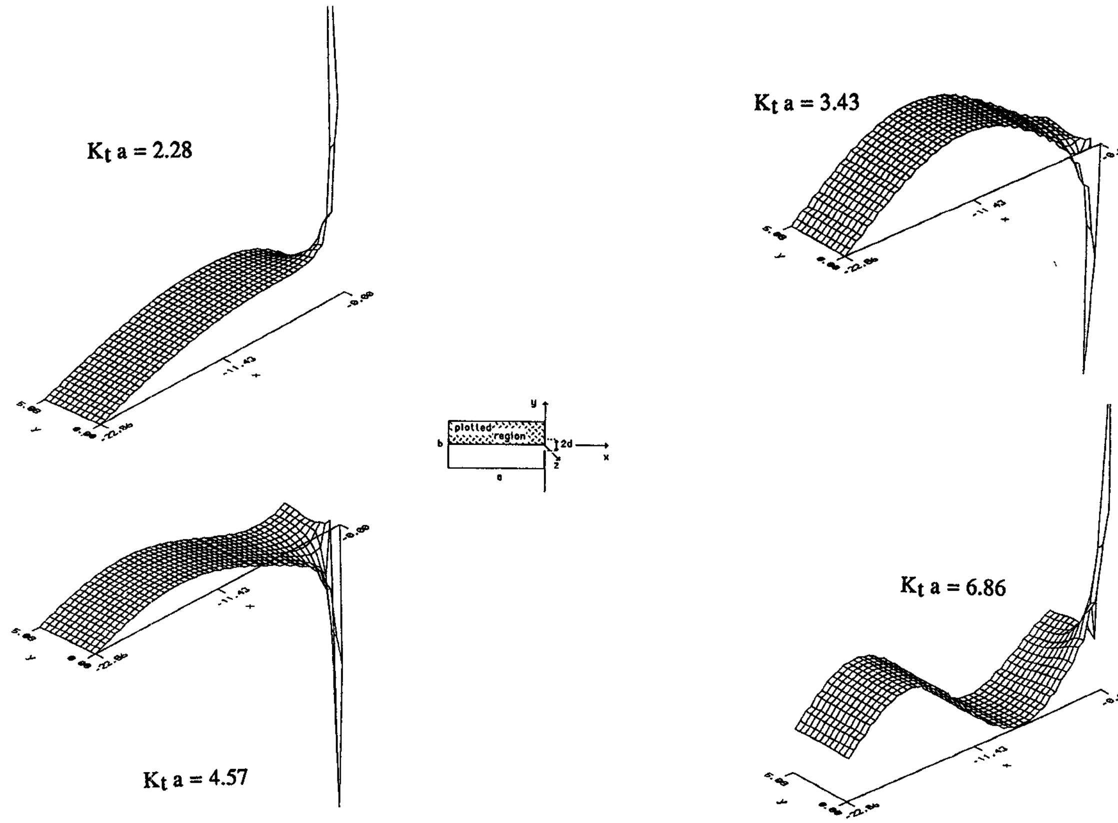

Fig. 4. Modal electric field of the slotted waveguide for different values of $k_t a$. The field is represented inside the guide, in the region $-a \leq x \leq 0$, $0 \leq y \leq b/2$, where a and b are the waveguide dimensions.

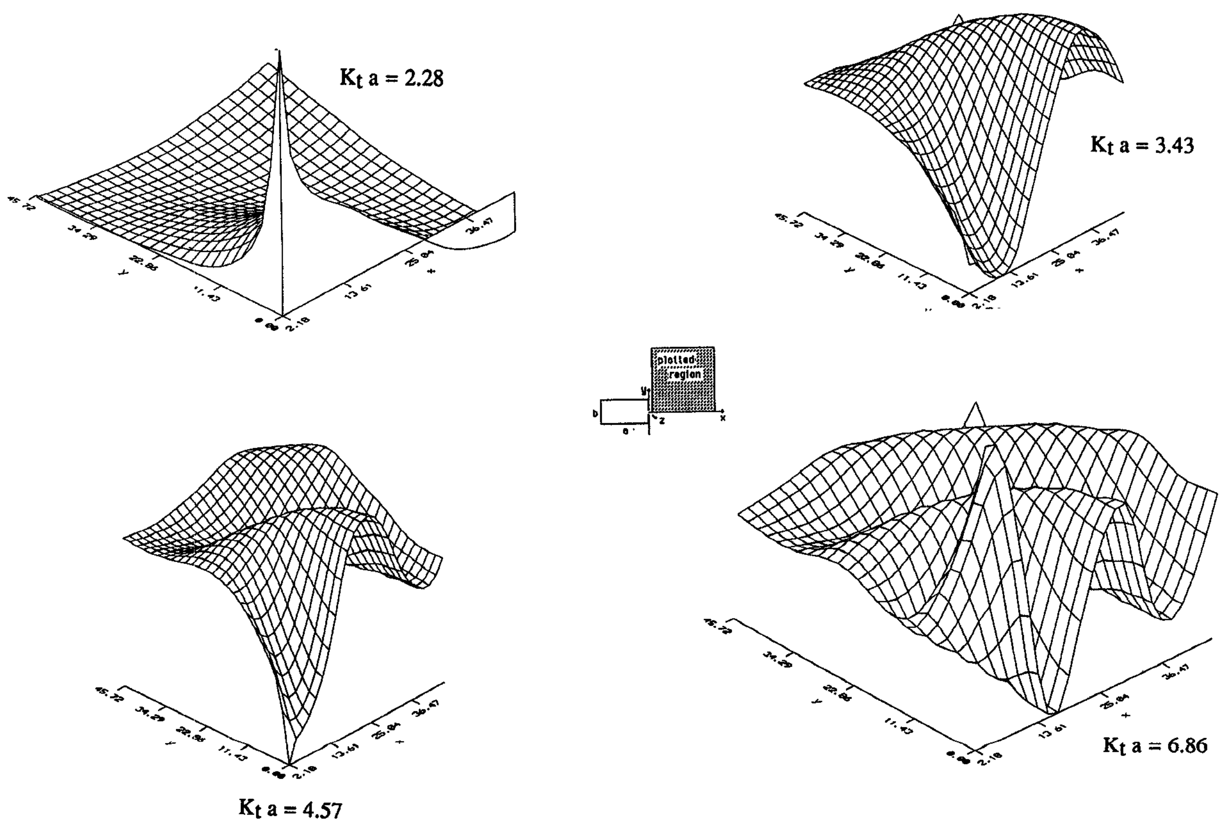

Fig. 5. Modal electric field of the slotted waveguide for different values of $k_t a$. The field is represented in the air region, for $0 < x \leq 2a$, $0 \leq y \leq 2a$, where a is the waveguide broad wall.

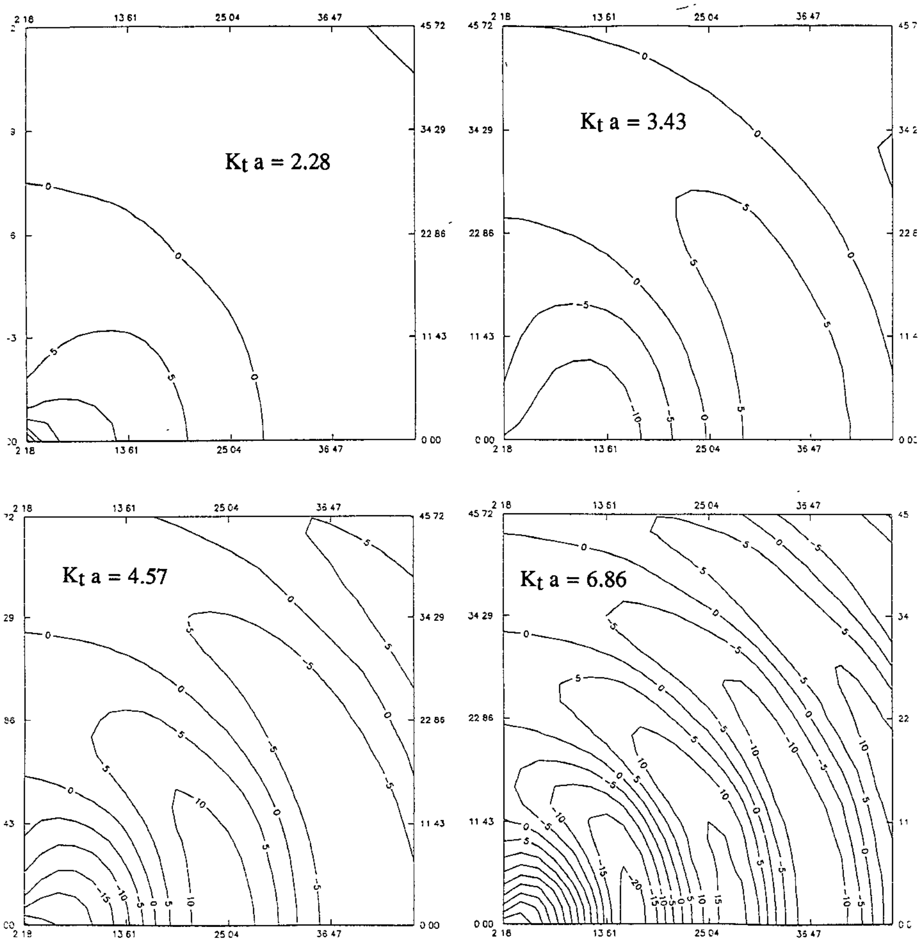

Fig. 6. Contour plots of the same modal fields represented in Fig. 5. The creation of a spherical wave is evident.

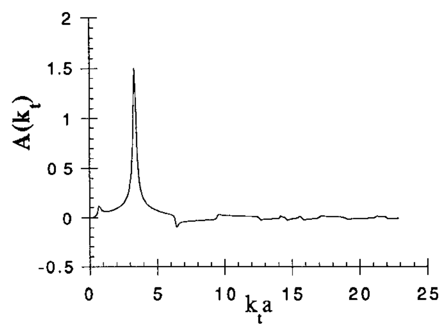

Fig. 7. Amplitude of the modal fields (continuous spectrum) excited by an incident TE_{10}. The slot width is $d/a = 1/22.86$.

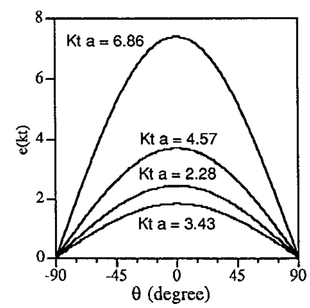

Fig. 9. Far field of the slotted waveguide on the plane xy obtained by steepest descent calculation. The angle θ is defined as in Fig. 1, and the electric field is reported on the y-axis.

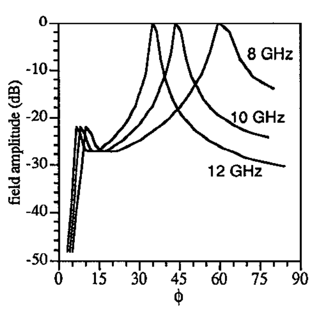

Fig. 8. Radiation patterns on the plane xz [ϕ = arctg (β/k_t)] for different frequencies. The slot width is $d/a = 1/22.86$.

The far field of the slotted guide, obtained by the steepest descent calculation of the Appendix as given by (A3), is plotted in Fig. 9 as a function of the angle Θ defined in Fig. 1.

Conclusions

In this contribution the continuum spectrum of a flange-backed rectangular waveguide slotted on its narrow wall has been derived. This part of the spectrum, which up to now has received little attention, describes radiation as simply, in principle, as the discrete spectrum of a classical guide describes any physical field in it. Numerical examples in the case of narrow slots have also been provided as an illustration of the theory. As an example of the simplicity of use of the above theory, the radiating properties of the junction between an ordinary guide and a slotted one have been investigated.

Appendix
Far Field of the Radiation Modes of Slotted Guide

The radiative part of a continuous mode is given by the first of the two integrals appearing in (3b). By substituting in this formula the following standard transformation

$$x = r\cos\theta \quad k_x = k_t\cos\theta$$
$$y = r\sin\theta \quad k_y = k_t\sin\theta \tag{A1}$$

we obtain

$$\begin{aligned} e_v &= \frac{N_v}{\sqrt{2\pi}}\left[\int_{-k_t}^{k_t}\tilde{E}_v\cos k_y y\cos k_x x\, dk_y \right.\\ &\quad \left. + \cot\alpha_v\int_{-k_t}^{k_t}\tilde{E}_v\cos k_y y\sin k_x x\, dk_y\right]\\ &= \frac{N_v k_t}{\sqrt{2\pi}}\left[\mathrm{Re}\int_C \tilde{E}_v e^{-\underline{k}_t\underline{r}}\cos\theta\, d\theta \right.\\ &\quad \left. + \cot\alpha_v\,\mathrm{Im}\int_C \tilde{E}_v e^{-\underline{k}_t\underline{r}}\cos\theta\, d\theta\right] \end{aligned} \tag{A2}$$

where C denotes the appropriate path in the complex θ plane defined by (A1) and

$$\underline{k}_t = (k_x, k_y); \quad \underline{r} = (x, y); \quad \underline{k}_t\cdot\underline{r} = k_x r\cos(\Theta - \theta);$$

Hence by saddler point integration at $\Theta = \theta$, we obtain the sought separation of radial and angular dependences, namely:

$$e_v \approx N_v\sqrt{\frac{k_t}{r}}\,\tilde{E}_v(\theta)\cos\theta\,\frac{\sin(k_t r - \pi/4 + \alpha_v)}{\sin\alpha_v} \tag{A3}$$

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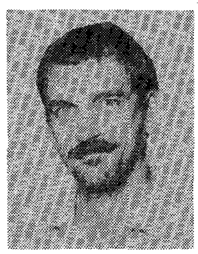
Tullio Rozzi (M'66–SM'74–F'90) was born in Italy in 1941. He obtained the degree of "dottore" in physics from Pisa University in 1965, the Ph.D. degree in electrical engineering from Leeds University in 1968, and the D.Sc. degree from the University of Bath in 1987.

From 1968 to 1978 he was a Research Scientist at the Philips Research Laboratories, Eindhoven, The Netherlands, having spent one year, 1975 at the Antenna Laboratory, University of Illinois, Urbana. In 1978 he was appointed to Chair of Electrical Engineering at University of Liverpool, U.K., and subsequently was appointed to the Chair of Electronics and Head of the Electronics Group at the University of Bath in 1981. From 1983 to 1986 he held the additional responsibility of Head of School of Electrical Engineering at Bath. Since 1986 he has held the Chair of Antennas at the Faculty of Engineering, University of Ancona, Italy, while remaining a Visiting Professor at Bath.

In 1975 Dr. Rozzi was awarded the Microwave Prize by the Microwave Theory and Techniques Society of the IEEE. He is a Fellow of the IEE (U.K.).



Mauro Mongiardo received the "Laurea" degree from the University of Rome and the Ph.D. from the University of Bath, U.K.

He is currently an Associate Professor at the University of Palermo, Italy. Since 1983 he has been engaged on microwave radiometry and inverse problems and in 1987 he was consultant of the Elettronica S.p.A. in the experimental validation of a four-channel radiometer developed for temperature retrieval of biological bodies.

In 1988 he was a recipient of a NATO-CNR research scholarship during which he was visiting researcher at the University of Bath (U.K.). In the summer of 1992 he was a visiting scientist at the University of Victoria, BC, Canada, working on time-domain analysis of MMIC. He is currently working in the modeling and computer-aided design of microwave and millimeter wave guiding structure and antennas, and in the modeling of discontinuities in MMIC and CPWs.